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Rethinking Fisher Effect with New Keynesian Phillips Curve

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Abstract

I modify and enlarge the simple Fisher equation by including different inflation dynamics and multiple common unobservable factors. I investigate the long runrelationship by carrying out a series of estimators and models that have been developed recently. I observe that augmentation of Fisher equation leads to provide the supportive evidence for the long run relationship between nominal interest rate and inflation.

Keywords: cross section dependence, common factors, Fisher Effect, panel unit root

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1. Introduction

The Fisher effect postulates the comovement between nominal interest rate and expected inflation. The Fisher relationship is often not supported by the empirical evidence. Mant studies, for instance Mishkin (1992) as a cornerstone, has rejected that the real interst rate exhibit a stationary process. Some other studies which have formally tested for cointegration and have not found any support for the cointegration relationship, are for instance, Evans and Lewis (1995), Engsted (1995), Koustas and Serletis (1999), Atkins and Serletis (2003) and Rapach (2003). On the other side, Westerlund (2008) recently tested the Fisher effect in a cointegrated panel of some OECD countries and could not reject the Fisher hypothesis.

In my study, I modify the well known simple Fisher equation with taking the inflation dynamics differently into account by using the econometric specification of the New Keynesian Phillips Curve (hereafter, NKPC). This type of change in Fisher equation provides to avoid the misspecification bias of the parameter estimations.

This paper also uses recent techniques in hetterogeneous panel data analysis to identify and acoount fort he unobserved common factors. The analysis comprises of two steps. In the first step, I search for the integration properties of the data using Pesaran, Smith and Yamagata (hereafter PSY, 2013) heterogeneous panel unit root tests based on the Dickey-Fuller type regression with multifactor error structure. In the second step, I estimate the relation between nominal interest rate and inflation with adding common factors into the regression. The parameters of the long run models are estimated via the approach of Dynamic OLS (DOLS) and Fully Modified OLS (FMOLS) for each country. Furthermore, cointegration analysis are performed through the residuals of these models by appliying Engle-Granger (hereafter EG, 1987) testing procedure. On the other side, augmented Fisher equation is also modelled in the form of ARDL(1,0) and Bounds testing procedure is applied to investigate the cointegration relationship. I compare the cointegration test results of the standard Fisher equation with the cointegration test results of augmented Fisher equation. I observe that that augmentation of the Fisher equation leads to obtain the significant long-run relationship for many of the countries.

2. Reexamination of Fisher Equation

The Fisherian theory of interest rates states that a permanent shock to inflation will lead to an equal change in nominal interest rates so that the real interest rate, i.e. the difference between the nominal interest rate and the expected inflation rate, is not affected by monetary shocks in the long run. Formally, the Standard Fisher equation can be set as follows

$$i_{it}^{m} = r_{it}^{m} + \beta_i E_t \pi_{it}^{m} \tag{1}$$

where i_{it}^{m} is the m-period nominal interest rate observed for country $i = 1,2, \dots, N$ at time $t = 1,2, \dots, T$; r_{it}^{m} is the m-period ex ante real interest rate and $E_t \pi_{it}^{m}$ is the expected m-period ahead rate of inflation. The Fisher effect cannot be investigated directly equation (1) ex ante real interest rate is assumed to be obtained through real factors which are changing relatively slowly over time such as technological change, risk aversion and time preference. Hence, it can be written as

$$r_{it}^{m} = \alpha_i + \varepsilon_{it} \tag{2}$$

where α_i is country specific effect, and ε_{it} is assumed to be stationary error term. Moreover, assuming rational expectations for expected inflation is to serve

$$\pi_{it} = E_t\pi_{it}^m + v_{it} \tag{3}$$

where v_{it} is a forecast error with $E_t v_{it} = 0$. Substituting (2) and (3) in (1) yields

$$i_{it} = \alpha_i + \beta_i\pi_{it} + u_{it} \tag{4}$$

where $u_{it} = \varepsilon_{it} + \beta_i v_{it}$ and they are stationary forecast errors.

However, inflation dynamics given in (3) may be interpreted as a misspecified model because macroeconomics literature proposes different inflation behaviours under the light of forward looking Phillips curve. I focus on the model what is currently used mostly, the New Keynesian Phillips Curve (NKPC) which has a popularity from the theoretical microfoundations and the success in empirical applications. An important property of NKPC is that inflation has a forward looking process which considers the expectations of future economic activity. Accordingly, NKPC is written as follows

$$\pi_{it} = \gamma_i E_t\pi_{i,t}^m + \lambda_i x_{it} + e_{it} \tag{5}$$

where instead of the next period inflation (i.e, m=1) $E_t\pi_{i,t+1} = \pi_{i,t-1}$ is usually taken into considerartion in the literature and x_{it} corresponds output gap here. (See Mavroedis et.al (2013) for details).

As long as the equation (5) is used for the equation (3), then, standard Fisher equation shall be as follows:

$$i_{it} = \alpha_i + \phi_i\pi_{it} + u_{it} \tag{6}$$

where $\phi_i = \beta_i/\gamma_i$ and $u_{it} = \varepsilon_{it} - \phi_i e_{it} - \phi_i\lambda_i x_{it}$.

Thus, we may state that the equation (4) as a misspecified model to test the Fisher effect when the observed variable x_{it} is located in the error structure. Moreover, there is not only one variable such as x_{it} which is omitted from the model but also it is possible that there may also some unobservable variables (or factors) whose are not covered with the regression examined in equation (4).

This study basically has an attempt to augment the Fisher equation to remove the misspecification bias and make comparisons between standard Fisher equation and the results from augmented Fisher equation in an empirical sense.

3. Econometric Models and Methods

As a first step, I have applied panel unit root tests allowing for the cross sectional dependence that have been advocated by PSY (2013). They have the tests based upon the model such that the error structure involves unobserved common multifactors. Some other tests proposed by Moon and Perron (2004) and Bai and Ng (2004) are also similar tests but they obtain the factor estimations from principal component analysis to remove the effect of cross section dependency. However, instead of using estimated common factors, PSY (2013) approach substitutes proxy variables for unobserved common factors. The significant parameters of these proxy variables may have explanatory power in a sense of economic theory although the value of factor loadings or the parameters of the factor variables do not have any such ability to make macroeconomic policy implications.

3.1 Modelling Individual Series

My first objective is to test whether unit root exists or not in a panel data framework. By following the methodology of PSY (2013), let y_{it} be the observation for each $i = 1,2, \dots, N$ and at time $t = 1,2, \dots, T$ and it has the process such that

$$\Delta y_{it} = \boldsymbol{\alpha_i}'\boldsymbol{d_t} + \beta_i y_{i,t-1} + u_{it} \tag{7}$$

where $y_{it} = (\pi_{it}, x_{it}\ or\ i_{it})$ and $\beta_i \leq 0$; d_t is a 2×1 vector as $d_t = (1, t)'$. Errors have the structure

$$u_{it} = \boldsymbol{\lambda_i}'\boldsymbol{f_t} + \varepsilon_{it} \tag{8}$$

where $\boldsymbol{f_t}$ is an $m \times 1$ vector of common factors and $\boldsymbol{\lambda_i}$ is associated vector of factor loadings and ε_{it}.

It is assumed that there are k-additional common observables $(\boldsymbol{z_{it}})$ which might be proxied for the multifactors $\boldsymbol{f_t}$. More specifically, additional regressors has the linear process such that

$$\Delta \mathbf{z}_{\boldsymbol{it}} = \boldsymbol{a_{iz}}\boldsymbol{d_t} + \boldsymbol{b_{iz}}\boldsymbol{f_t} + \boldsymbol{e_{it}} \tag{9}$$

where $\mathbf{z}_{\boldsymbol{it}} = (\mathrm{z_{i1t}, z_{i2t}, \dots, z_{ikt}})'$, $\mathbf{b_{iz}} = (\mathrm{b_{i1}, b_{i2}, \dots, b_{ik}})'$and $\mathbf{a_{iz}} = (\mathrm{a_{i1}, a_{i2}, \dots, a_{ik}})'$. Notice here that $e_{it} \sim I(0)$.

Furthermore, the equations (7),(8) and (9) are combined in an appropriate way and shown with a matrix form as follows

$$\Delta \boldsymbol{s_{it}} = \boldsymbol{A_i}\boldsymbol{d_t} + \boldsymbol{B_i} y_{i,t-1} + \boldsymbol{C_i'}\boldsymbol{f_t} + \boldsymbol{E_{it}} \tag{10}$$

where $\Delta \boldsymbol{s_{it}} = (\Delta \mathrm{y}_{it}, \Delta \boldsymbol{z_{it}'})'$, $\boldsymbol{A_i} = (\alpha_i, \boldsymbol{a_{iz}'})'$, $\boldsymbol{B_i} = (\beta_i, \mathbf{0}'\ )'$, $\boldsymbol{C_i} = (\boldsymbol{\lambda}_i, \boldsymbol{b_{iz}})$ and $\boldsymbol{E_{it}} = (\varepsilon_{it}, \boldsymbol{e_{it}'})'$.
Note that $\Delta \boldsymbol{s_{it}}, \boldsymbol{A_i}, \boldsymbol{B_i}$and $\boldsymbol{E_{it}}$ are $(k + 1) \times 1$ vectors whereas , $\boldsymbol{C_i}$ is $m \times (k + 1)$ matrix.
Averaging (10) across individuals $(i = 1,2, \dots N)$, I arrive

$$\overline{\Delta \boldsymbol{s}}_{\boldsymbol{t}} = \overline{\boldsymbol{A}}\boldsymbol{d_t} + \overline{\boldsymbol{B}}\,\bar{y}_{t-1} + \overline{\boldsymbol{C}}'\boldsymbol{f_t} + \overline{\boldsymbol{E}}_{\boldsymbol{t}} \tag{11}$$

Additionally, following Pesaran (2006) approach, common factors can be illustrated with

$$\boldsymbol{f_t} = (\,\overline{\boldsymbol{C}}\overline{\boldsymbol{C}}')^{-\mathbf{1}}\overline{\boldsymbol{C}}[\ \overline{\Delta \boldsymbol{s}}_{\boldsymbol{t}} - \overline{\boldsymbol{A}}\boldsymbol{d_t} - \overline{\boldsymbol{B}}\,\bar{y}_{t-1} - \overline{\boldsymbol{E}}_{\boldsymbol{t}}\ ] \tag{12}$$

and since $\overline{\boldsymbol{E}}_{\boldsymbol{t}} \overset{\mathrm{N}}{\rightarrow} 0$, then

$$\boldsymbol{f_t} - (\,\overline{\boldsymbol{C}}\overline{\boldsymbol{C}}')^{-\mathbf{1}}\overline{\boldsymbol{C}}[\ \overline{\Delta \boldsymbol{s}}_{\boldsymbol{t}} - \overline{\boldsymbol{A}}\boldsymbol{d_t} - \overline{\boldsymbol{B}}\,\bar{y}_{t-1} - \overline{\boldsymbol{E}}_{\boldsymbol{t}}\ ] \overset{\boldsymbol{N}}{\rightarrow} \mathbf{0} \tag{13}$$

Hence, the linear combinations of $\overline{\Delta \boldsymbol{s}_t}, \boldsymbol{d_t}$and $\bar{y}_{t-1}$ can be used as proxies for the common factors $\boldsymbol{f_t}$ in the case of N → ∞**.** Under the light of the equations derived above, panel unit root tests can be performed with the following cross-sectionally augmented regression

$$\Delta y_{it} = \boldsymbol{\alpha}_i^{'}\boldsymbol{d_t} + \beta_i y_{i,t-1} + \theta_i \bar{y}_{t-1} \quad + \boldsymbol{\gamma_i}'\overline{\Delta \boldsymbol{s}_t} + \xi_{it} \qquad (14)$$

And corresponding t-ratio statistics of $\widehat{\beta}_i$:

$$t_{i,NT} = \frac{(\sqrt{T-2k-5})\boldsymbol{\Delta y_i'}\boldsymbol{\bar{M}}\boldsymbol{y_{i,-1}}}{(\boldsymbol{\Delta y_i'}\boldsymbol{\bar{M}}\boldsymbol{\Delta y_i})^{1/2}(\boldsymbol{y_{i,-1}'}\boldsymbol{\bar{M}}\boldsymbol{y_{i,-1}})^{1/2}} \qquad (15)$$

where $\bar{M} = I_T - \overline{\boldsymbol{W}}(\overline{\boldsymbol{W}}'\overline{\boldsymbol{W}})^{-1}\overline{\boldsymbol{W}}'$ and $\overline{\boldsymbol{W}}$=($\boldsymbol{\Delta \bar{s}_t}'$, $\boldsymbol{d_t'}$, $\bar{y}_{t-1}$')' .

The panel unit root test can now be calculated from the average of the t-ratio

$$CIPS_{NT} = N^{-1}\sum_{i=1}^{N} t_i \qquad (16)$$

which is the cross sectionally augmented version of the IPS test of Im et. al (2003).

Furthermore, I also apply the test of Sargan and Bhargava (1983) which has been firstly adopted by Bai and Ng (2010) and PSY (2013) in the framework of panel data. Under the null hypothesis of $\boldsymbol{B_i} = 0$, then the model (10) shall be rewritten as

$$\Delta \boldsymbol{s_{it}} = \boldsymbol{A_i d_t} + \boldsymbol{C_i'f_t} + \boldsymbol{E_{it}} \qquad (17)$$

and taking the average of this regression with respect to the cross section units then factors converge to some fixed regressors since N → ∞ as represented below

$$\boldsymbol{f_t} - (\,\overline{\boldsymbol{C}}\overline{\boldsymbol{C}}')^{-1}\overline{\boldsymbol{C}}[\;\overline{\Delta \boldsymbol{s}_t} - \overline{\boldsymbol{A}}\boldsymbol{d_t}\;] \xrightarrow{\boldsymbol{N}} \boldsymbol{0} \qquad (18)$$

Notice again that $\overline{\boldsymbol{E}}_t \xrightarrow{N} 0$. Thus, factors can be proxied with the linear combinations of $\overline{\Delta \boldsymbol{s}_t}$ and $\boldsymbol{d_t}$ **.** Sargan-Bhargava (SB) statistic is to be obtained through the following regression

$$\Delta y_{it} = \boldsymbol{\alpha}_i^{'}\boldsymbol{d_t} + \boldsymbol{\gamma_i}'\overline{\Delta \boldsymbol{s}_t} + \epsilon_{it} \qquad (19)$$

Cross sectionally augmented SB statistic is calculated from the OLS residuals of the above model such that

$$CSB_{i,NT} = T^{-2}\;\sum_{t=1}^{T}\,\hat{u}_{it}^2/\,\hat{\sigma}_i^2 \qquad (20)$$

where $\hat{u}_{it}^2 = \sum_{j=1}^{t}\hat{\epsilon}_{ij}$ and $\hat{\sigma}_i^2 = (T-k-1)^{-1}\sum_{t=1}^{T}\widehat{\epsilon_{it}}^2$.

The panel version of $CSB_{i,NT}$ statistics is computed from the formulation given as

$$CSB_{NT} = N^{-1}\sum_{i=1}^{N} CSB_{i,NT} \qquad (21)$$

The empirical results for all these statistics are served in the empirical application part.

3.2 Long Run Models and Tests

In this section, I introduce the cointegration models which are to be useful for the analysis of Fischer effect. Models are designed appropriately for dynamic OLS estimation (DOLS), Fully Modified OLS (FMOLS) and Autoregressive Distributed Lag Model (ARDL) based on the OLS estimation for each individual.

The models are extended via including proxies for unobserved factors so that they are remarkably different standard dynamic cointegration and ARDL based error correction models. In a general set up, the fisher equation with examining the inflation dynamics as in New Keynesian Phillips Curve, can be written as follows

$$i_{it} = \boldsymbol{a_i}'\boldsymbol{d_t} + b_i\pi_{it} + \lambda_i'\boldsymbol{f_t} + \varepsilon_{it} \quad (22)$$
$$\Delta\pi_{it} = \boldsymbol{\alpha_i'}\boldsymbol{d_t} + \beta_i\pi_{i,t-1} + \theta_i x_{it} + \boldsymbol{b_{i1}'}\boldsymbol{f_t} + e_{it} \quad (23)$$
$$\Delta x_{it} = \boldsymbol{\tau_i'}\boldsymbol{d_t} + \boldsymbol{b_{i2}'}\boldsymbol{f_t} + u_{it} \quad (24)$$

Notice here that, under the light of NKPC approach, inflation dynamics is set out $\pi_{it} = \boldsymbol{\alpha_i'}\boldsymbol{d_t} + \gamma_i\pi_{i,t-1} + \theta_i x_{it} + \boldsymbol{b_{i1}'}\boldsymbol{f_t} + e_{it}$ but since $\beta_i = \gamma_i - 1$, then NKPC is simply written as in equation (23).

To find the proxies for the multifactor structure the system of equations can be set in matrix notation as

$$\boldsymbol{z_{it}} = \boldsymbol{A_i'}\boldsymbol{d_t} + \boldsymbol{B_i}\pi_{it} + \boldsymbol{C_i}\,\pi_{i,t-1} + \boldsymbol{D_i}x_{it} + \boldsymbol{\Gamma_i}\mathbf{f_t} + \boldsymbol{E_{it}} \quad (25)$$

where $\boldsymbol{A_i} = (a_i, \alpha_i, \tau_i)'$, $\boldsymbol{B_i} = (b_i, 0, 0)'$, $\boldsymbol{D_i} = (0, \theta_i, 0)'$, $\boldsymbol{\Gamma_i} = (\lambda_i, b_{i1}, b_{i2})'$ and $\boldsymbol{z_{it}} = (i_{it}, \Delta\pi_{it}, \Delta x_{it})'$.

Hence, after being taken average of (25) with respect to cross section, $\mathbf{f_t}$ will be left alone as follows

$$\boldsymbol{f_t} = (\bar{\boldsymbol{\Gamma}}'\bar{\boldsymbol{\Gamma}})^{-1}\bar{\boldsymbol{\Gamma}}'[\,\bar{\boldsymbol{z}}_t - \bar{\boldsymbol{A}}\boldsymbol{d_t} - \bar{\boldsymbol{B}}\,\overline{\pi_t} - \bar{\boldsymbol{C}}\bar{\pi}_{t-1} - \bar{D}\bar{x}_t - \bar{E}\,] \quad (26)$$

since $\bar{\mathbf{E}} \xrightarrow{N} 0$, then

$$\boldsymbol{f_t} - (\bar{\boldsymbol{\Gamma}}'\bar{\boldsymbol{\Gamma}})^{-1}\bar{\boldsymbol{\Gamma}}'[\,\bar{\boldsymbol{z}}_t - \bar{\boldsymbol{A}}\boldsymbol{d_t} - \bar{\boldsymbol{B}}\,\overline{\pi_t} - \bar{\boldsymbol{C}}\bar{\pi}_{t-1} - \bar{D}\bar{x}_t\,] \xrightarrow{N} \mathbf{0} \quad (27)$$

Accordingly, there are six proxy variables available to be added to the Fisher equation which may lead to study with very large model and since the number of observations are limited as in my applications then the parameter estimations and corresponding tests may not be reliable in a statistical sense. Therefore, to reduce the number of proxies (i.e dimension reduction), I first reconsider the equations (23) and (24) with a matrix form as follows

$$\boldsymbol{\Delta p_{it}} = \boldsymbol{K_i}\boldsymbol{d_t} + \boldsymbol{L_i}\pi_{i,t-1} + \boldsymbol{M_i}x_{it} + \boldsymbol{\Lambda_i}\mathbf{f_t} + \boldsymbol{V_{it}} \quad (28)$$

where $\boldsymbol{\Delta p_{it}} = (\Delta\pi_{it}, \Delta x_{it})'$, $\boldsymbol{K_i} = (\alpha_i, \tau_i)'$, $\boldsymbol{L_i} = (\beta_i, 0)'$, $\boldsymbol{M_i} = (\theta_i, 0)'$, $\boldsymbol{\Lambda_i} = (\boldsymbol{b_{i1}}, \boldsymbol{b_{i2}})'$ and $\boldsymbol{V_{it}} = (e_{it}, u_{it})'$.

In a second step, again taking average of (28) over the cross section is to serve the common factor equation as follows

$$\boldsymbol{f_t} - (\overline{\boldsymbol{\Lambda}}'\overline{\boldsymbol{\Lambda}})^{-1}\overline{\boldsymbol{\Lambda}}' \; [\overline{\boldsymbol{\Delta p_t}} - \overline{\boldsymbol{K}}\boldsymbol{d_t} - \overline{\boldsymbol{L}}\bar{\pi}_{t-1} - \overline{\mathbf{M}}\bar{x}_t] \xrightarrow{N} \mathbf{0} \qquad (29)$$

since $\overline{\mathbf{V}} \xrightarrow{N} \infty$. Hence, $\overline{\boldsymbol{\Delta p_t}}, \bar{\pi}_{-1}$ and $\bar{x}_t$ are the variables that will be injected as proxies into the inflation equation. In a final step, I run the regression for each country

$$\Delta\pi_{it} = \boldsymbol{\alpha_i' d_t} + \beta_i\pi_{i,t-1} + \theta_i x_{it} + b_{i1}\overline{\Delta\pi}_t + b_{i2}\bar{x}_t + b_{i3}\overline{\Delta x}_t + b_{i4}\bar{\pi}_{t-1} + e_{it} \qquad (30)$$

and then prediction values of $\Delta\pi_{it}$, i.e $\widehat{\Delta\pi}_{it}$, is computed and $\overline{\widehat{\Delta\pi}}_t = N^{-1}\sum_{i=1}^{N}\widehat{\Delta\pi}_{it}$ is plugged into the Fisher equation instead of the four proxy variables given in the above regression (30) to reduce the dimension of the regression (22) and overcome the degrees of freedom problem in that equation. Since the equations (23) and (24) are dropped from the system of equations, the resultant Fisher equation can be derived only from the equation (22) and the associated factors is going to handle after its cross section average is computed appropriately. It is shown as the following

$$\boldsymbol{f_t} - (\bar{\boldsymbol{\lambda}}\bar{\boldsymbol{\lambda}}')^{-1}\bar{\boldsymbol{\lambda}}[\bar{\iota}_t - \bar{\boldsymbol{\alpha}}\boldsymbol{d_t} - \bar{b}\bar{\pi}_t] \xrightarrow{N} \mathbf{0} \qquad (31)$$

since $\bar{\varepsilon}_t \xrightarrow{N} 0$. Thus, equation (22) serves two proxy variables $\bar{\iota}_t$ and $\bar{\pi}_t$ for unobserved factors and the additional variable $\overline{\widehat{\Delta\pi}}_t$ stemmed from the equations (23) and (24) is also added to the Fisher equation to arrive the *benchmark* model as

$$i_{it} = a_i d_t + b_i\pi_{it} + \lambda_{i1}\overline{\widehat{\Delta\pi}}_t + \lambda_{i2}\bar{\iota}_t + \lambda_{i3}\bar{\pi}_t + \varepsilon_{it} \qquad (32)$$

and this is named with *Factor Augmented Fisher* equation.

4. Empirical Findings

In this section, I first start to give the results from panel CIPS and CSB tests as described in the previous section in order to ascertain the order of integration for the variables nominal interest rate, inflation and output gap. In the second step, cointegration test of Engle-Granger (1987) and Bound testing procedure of Pesaran et.al (2001) are put into process by modifying the benchmark model of (32).

I employ yearly data taken from OECD statistics database. The sample includes 18 OECD countries over the period 1985-2019. The determination of the countries and the time frequency of the sample period depend upon the data availability. For instance, it is hard to get

monthly or quarterly output gap data for each country. Moreover, I have used ex post observed inflation rate for expected inflation rate.
In testing of panel unit roots with CIPS and CSB tests, I have computed t-ratio statistics from the regression (14) with involving two variables from the set of $\{\overline{\Delta\pi_t}, \overline{\Delta x_t}\ or\ \overline{\Delta\iota_t}\} \in \overline{\boldsymbol{\Delta s_t}}$ and one variable from from $\{\bar{\pi}_{t-1}, \bar{x}_{t-1}\ or\ \bar{\iota}_{t-1}\} \in \bar{\mathbf{y}}_{\mathbf{t-1}}$. Hence, in my setting $m_{max} = 4$ which might be examined enough to explain variations in most macroecnomic variables as proposed in the works of Stock and Watson (2002) and Eickmeier (2009).

Table 1. Panel Unit Root Tests for Interest Rate

Countries	*CIPS Statistics* $(\boldsymbol{t-statistics})$		*CSB Statistics* $(\boldsymbol{t-statistics})$	
	Only Intercept	*Intercept and Trend*	*Only Intercept*	*Intercept and Trend*
Australia	-0.67	-0.61	0.14	1.16
Austria	-0.82	-0.85	0.17	2.16
Belgium	-0.61	-0.87	0.14	1.91
Canada	-0.63	-1.18	0.06	0.46
Denmark	-1.16	-2.46	0.12	2.09
Finland	-0.54	-0.42	0.13	1.36
France	-1.56	-1.38	0.15	2.19
Iceland	-2.31	-1.82	0.31	0.54
Italy	-1.42	-2.50	0.22	2.62
Korea	-1.43	-2.65	0.09	1.26
Netherland	-0.12	-1.03	0.08	0.89
Norway	-1.21	-1.42	0.09	1.67
Portugal	-0.78	-0.54	0.13	2.28
Spain	-2.04	-1.88	0.16	2.22
Sweden	-1.55	-2.44	0.16	1.90
Switzerland	-0.26	-1.15	0.07	0.35
UK	-0.07	-0.81	0.07	1.18
US	-1.74	-2.33	0.08	0.21
Averages	-1.05	-1.46	0.13	1.46***

Note: (*),(**),(***) are significance levels of 1%,5% and 10% respectively.

I present the results for each country in the Tables 1 to 3. I have supplied the results for the cases with only intercept term and intercept plus linear trend terms. The critical values produced in the article of PSY(2013) is considered in my interpretations.

As can be seen from the average values of the statistics, the CIPS test strongly accepts the null of panel nonstationarity whereas the CSB statistic reject the null of stationarity for the variables interest rate and output gap. On the other hand, inflation rate has different tendency that all the test outcomes except CSB statistic with only intercept term are supporting the stationarity for 1% and 5% significance levels. Therefore, results for inflation rate yield somewhat mixed results.

Table 2. Panel Unit Root Tests for Output Gap

Countries	***CIPS Statistics*** $(\boldsymbol{t-statistics})$		***CSB Statistics*** $(\boldsymbol{t-statistics})$	
	Only Intercept	*Intercept and Trend*	*Only Intercept*	*Intercept and Trend*
Australia	-2.19	-2.30	0.05	0.08
Austria	-2.07	-1.95	0.05	0.06
Belgium	-1.61	-1.28	0.23	1.30
Canada	-1.57	-1.17	0.09	0.10
Denmark	-2.15	-2.27	0.10	0.11
Finland	-1.36	-1.54	0.10	0.17
France	-2.18	-2.10	0.36	0.62
Iceland	-2.07	-2.52	0.05	0.04
Italy	-2.18	-2.35	0.13	0.10
Korea	-2.91	-2.91	1.30	2.25
Netherland	-1.24	-1.68	0.05	0.05
Norway	-2.31	-2.22	0.14	0.46
Portugal	-2.28	-3.42	0.43	0.63
Spain	-2.64	-3.14	0.22	0.25
Sweden	-2.65	-2.64	0.10	0.08
Switzerland	-0.98	-0.41	0.13	0.17
UK	-2.51	-2.43	0.05	0.11
US	-1.49	-2.66	0.05	0.05
Average	-2.02	-2.66	0.22***	0.37***

Note: (*),(**),(***) are significance levels of 1%,5% and 10% respectively

Table 3. Panel Unit Root Tests for Inflation Rate

Countries	***CIPS Statistics*** $(\boldsymbol{t-statistics})$		***CSB Statistics*** $(\boldsymbol{t-statistics})$	
	Only Intercept	*Intercept and Trend*	*Only Intercept*	*Intercept and Trend*
Australia	-1.96	-1.89	0.10	0.13
Austria	-3.48	-3.39	0.11	0.57
Belgium	-5.54	-5.57	0.15	0.75
Canada	-3.94	-4.02	0.05	0.10
Denmark	-1.58	-1.78	0.09	0.60
Finland	-0.17	-0.19	0.19	0.96
France	-5.85	-5.74	0.31	1.38
Iceland	-1.25	-0.74	0.35	1.18
Italy	-4.42	-6.23	0.25	1.57
Korea	-3.79	-4.46	0.05	0.05
Netherland	-2.98	-3.11	0.04	0.06
Norway	-1.60	-1.56	0.15	0.63
Portugal	-4.93	-5.11	0.40	1.74
Spain	-5.03	-5.33	0.17	1.07
Sweden	-3.28	-3.26	0.09	0.22

Note: (*),(**),(***) are significance levels of 1%,5% and 10% respectively.

As an illustration of my benchmark model, I consider Engle-Granger (EG) and Bounds cointegration tests that were applied for each country and I compare the empirical findings of the model having common factors with the results from the model without common factors. I have first modified benchmark model by adding one lag and lead variables and even their averages to obtain DOLS based estimation of the errors. In the second step, I have computed EG τ-statistics for the residuals and presented the relevant results in Table 4.
I observe that augmented Fisher equation with involving common factors have tendency to support the Fisher effect. For instance, Belgium, France, Finland and Norway cannot reject the Fisher hypothesis when the factor augmented model is recognized empirically. In a similar manner, the model with including only intercept term for Sweden, UK and US has validated the Fisher effect whereas this conclusion cannot be proposed for the results based on the model without common factors.

On the other hand, FMOLS estimation is performed over the benchmark model and its residuals are used for testing whether there is evidence for long run relationship or not with EG cointegration test. The results given in Table 5 demonstrate that there are rejections of the null of no cointegration for many countries when the EG test is evaluated for the factor augmented residuals. There are sharp changes in test statistics results such that Australia, Belgium, Denmark, France, Finland, Netherland and UK do not show any supportive evidence in the direction of the existence of long run relationship when common factors are excluded from the benchmark model, whereas all these countries have the evidences in favour of the Fisher effect as long as the benchmark model is run and perform the tests over their residuals.

Table 4. Engle Granger Test Results Based on DOLS Estimation

Countries	***Without Common Factors***	***Without Common Factors***	***With Common Factors***	***With Common Factors***
	Only Intercept	*Intercept and Trend*	*Only Intercept*	*Only Intercept*
Australia	-3.24*	-4.05**	-5.52***	-3.90*
Austria	-1.49	-4.78*	-3.79**	-3.67
Belgium	-1.31	-3.49	-4.70***	-4.56**
Canada	-3.04	-3.98*	-5.63***	-5.75***
Denmark	-2.25	-3.90*	-4.11**	-4.69**
Finland	-2.20	-2.20	-3.60**	-3.75*
France	-2.27	-2.27	-3.78**	-4.56**
Iceland	-4.66***	-3.99*	-5.51***	-5.52***
Italy	-3.67**	-3.06	-5.50***	-5.05***
Korea	-3.15*	-2.08	-3.05	-2.45
Netherland	-0.92	-2.70	-3.02	-3.67
Norway	-1.36	-2.97	-3.95**	-4.17**
Portugal	-3.41*	-2.30	-3.34*	-3.16
Spain	-3.42*	-3.04	-3.91**	-3.92*
Sweden	-1.66	-5.50***	-5.76***	-6.08***
Switzerland	-4.75***	-6.03***	-5.72***	-5.79***
UK	-1.79	-4.52**	-3.57**	-3.70*
US	-3.00	-4.37**	-4.10**	-4.15

Note: (*),(**),(***) are significance levels of 1%,5% and 10% respectively.

Due to the possibility of stationary inflation rates, ARDL method is also utilized in this study. In contrast to the EG test, ARDL can be recognized to test for a level relationship for variables I(0) or I(1) as well as for a mixture of I(0) and I(1) variables. Bounds test is performed through the following modified benchmark model

$$\Delta i_{it} = a_i d_t + \beta_{1i}\pi_{i,t-1} + \beta_{2i} i_{i,t-1} + \theta_{1i}\Delta i_{i,t-1} + \theta_{2i}\Delta\pi_{it} + \lambda_{i1}\overline{\widehat{\Delta\pi}}_t + \lambda_{i2}\overline{\iota_{t-1}} + \lambda_{i3}\overline{\Delta\iota_t} + \varepsilon_{it} \quad (33)$$

Notice here that $\overline{\widehat{\Delta\pi}}_t$ captures the proxy variables $\bar{\pi}_{-1}$ and $\overline{\Delta\pi}_t$ as previously discussed in section 3.2. Table 6 supplies the empirical results for the Bounds test with/without common factor specifications.

The findings from the Table 6 report that there is considerable evidence against the long-run Fisher effect if one believes that usual (without common factor) Fisher equation is enough to represent the relationship between nominal interest rate and inflation rate. However, the results from the application of Bounds test regarding of the model (33) imply that we cannot reject the cointegration relationship between these variables in most of the countries. The story of the results illustrated in Table 6 is similar to the results of both Tables 4 and 5 in a sense that Austria, Belgium, Canada, Denmark, Netherland and US tend to show supportive evidence for the long run relationship between interest rate and inflation when the model (33) is taken into account for Bounds test analysis.

Table 5. Engle Granger Test Results Based on FMOLS Estimation

Countries	*Without Common Factors*	*Without Common Factors*	*With Common Factors*	*With Common Factors*
	Only Intercept	*Intercept and Trend*	*Only Intercept*	*Only Intercept*
Australia	-3.73	-2.66	-3.83**	-4.66**
Austria	-4.62**	-1.90	-4.93***	-5.17***
Belgium	-2.77	-1.54	-3.64**	-3.49
Canada	-3.94*	-2.88	-4.66***	-4.68***
Denmark	-2.55	-2.25	-3.93**	-3.91*
Finland	-2.58	-2.28	-4.07**	-5.30***
France	-2.20	-2.25	-4.57***	-4.59***
Iceland	-4.57**	-4.28***	-5.41***	-5.40***
Italy	-2.94	-3.31*	-4.79***	-4.39**
Korea	-2.39	-3.85**	-4.03**	-2.07
Netherland	-2.53	-1.15	-4.18***	-4.62**
Norway	-3.91*	-2.35	-4.36***	-4.61**
Portugal	-3.03	-2.12	-2.31	-2.33
Spain	-3.49	-4.05**	-4.16***	-4.24**
Sweden	-5.14***	-2.95	-4.86***	-5.54***
Switzerland	-5.56***	-4.90***	-6.41***	-6.50***
UK	-3.15	-1.97	-3.17***	-3.19
US	-4.63**	-3.29*	-3.32*	-3.65

Note: (*),(**),(***) are significance levels of 1%,5% and 10% respectively.

Table 6. Bounds Test Results Based on ARDL(1,0) Model

Countries	*Without Common Factors*	*Without Common Factors*	*With Common Factors*	*With Common Factors*
	Only Intercept	*Intercept and Trend*	*Only Intercept*	*Only Intercept*
Australia	3.49	5.16**	4.02*	4.80*
Austria	1.77	2.00	3.65*	3.50
Belgium	1.37	2.30	7.37***	6.60**
Canada	1.14	3.14	5.00**	4.77*
Denmark	2.33	2.48	6.38***	6.15**
Finland	4.74**	5.63**	8.49***	7.65***
France	1.67	2.46	3.12 (IC)	3.41
Iceland	1.54	2.55	3.97*	4.24(IC)
Italy	4.60**	4.10	9.85***	10.37***
Korea	1.54	1.71	1.23	1.47
Netherland	1.14	2.53	4.94**	4.89*
Norway	2.20	4.79***	4.85**	5.26**
Portugal	17.40*	15.50*	17.60***	14.60***
Spain	4.69**	4.44	9.90***	10.61***
Sweden	4.29	12.50*	8.05***	8.54***
Switzerland	3.47	8.42*	10.54***	8.79***
UK	0.64	3.75	3.22(IC)	3.75
US	1.37	1.88	3.64*	4.76*

Note: (*),(**),(***) are significance levels of 1%,5% and 10% respectively.

5. Conclusion

This paper has analyzed Fisher effect in nonstationary panel data framework. I have proposed augmented Fisher equation based upon recently developed methods for nonstationary panels where the cross section units are correlated through common factor structure. Different models were implemented and cointegration tests are performed over these distinguished models. The results imply that it is essential to construct correctly specified models to make statistical inference whether there is cointegration between the variables or not.

I have found that augmented fisher model usually captures the long run relation in most of the countries. However, this implication is not proposed for the models without paying attention to unobserved factors in the error structures.

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