SCIENTIFIC LETTERS

Econometrics Letters Volume (1), 1, 2014, June.

Nonlinearity and Smooth Breaks in Unit Root Testing

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Abstract

We develop unit root tests that allow under the alternative hypothesis for a smooth transition between deterministic linear trends, around which stationary asymmetric adjustment may occur by employing exponential smooth transition auto-regressive (ESTAR) models The small sample properties of the newly developed test are briefly investigated and an application for investigating the PPP hypothesis for Argentina is provided.

Keywords: Smooth Break; Nonlinear Unit Root Test; PPP.
JEL Codes: C12; C22; O47.

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1. Introduction

In this study we have developed a test for the unit root null hypothesis by combining the methodologies developed by Kapetanois et al (2003) (henceforth KSS) and Leybourne, Newbold and Vougas (1998) (henceforth LNV). KSS (2003) employ exponential smooth transition autoregressive (ESTAR) models to propose tests of the null hypothesis of a unit root that allow under the alternative hypothesis for stationary nonlinear adjustment towards a fixed mean. Thus, we extent the KSS tests to the case of a nonlinear attractor[3].

Section 2 of this paper develops the proposed test statistics and represents their critical values. Section 3 provides the small sample performance of the proposed test in comparison with the power of the ADF, LNV, Sollis, KSS and EG tests. Section 4 presents the application of our aforementioned tests to the PPP hypothesis.

2. The model and testing framework

Let y_t be a changing trend function with smooth transition on the time domain $t = 1, 2, ..., T$.

$$y_t = \alpha + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t \tag{1}$$

$$y_t = \alpha + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t \tag{2}$$

$$y_t = \alpha + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \beta_2 t S_t(\gamma, \tau) + \varepsilon_t \tag{3}$$

where ε_t is a zero mean $I(0)$ process and $S_t(\gamma, \tau)$ is logistic smooth transition function, based on a sample of size T and N,

$$S_t(\gamma, \tau) = \left[1 + \exp\{-\gamma(t - \tau T)\}\right]^{-1}, \quad \gamma > 0 \tag{4}$$

In this modeling strategy, the structural change is modeled as a smooth transition between different regimes rather than an instantaneous structural break as in Leybourtne *et al.* (1996). The transition function $S_t(\gamma, \tau)$ is a continuous function bounded between 1 and 0. Thus the STR model can be interpreted as regime-switching model that allows for two regimes, associated with the extreme values of the transition function, $S_t(\gamma, \tau) = 0$ and $S_t(\gamma, \tau) = 1$, whereas the transition from one regime to the other is gradual. The parameter γ determines the smoothness of the transition, and thus, the smoothness of transition from one regime to the other. The two regimes are associated with small and large values of the transition variable $s_t = t$ relative to the threshold $c = \tau$. For the large values of γ, $S_t(\gamma, \tau)$ passes through the interval $(0,1)$ very rapidly, and as γ approaches $+\infty$ this function changes value from 0 to 1 instantaneously at time $t = \tau T$. Therefore, if we assume that ε_t is a $I(0)$ process with zero mean, then in model 1 y_t is a stationary process around a mean which changes from initial value α_1 to final value $\alpha_1 + \alpha_2$. Leybourne *et al.* (1996) also give similar conditions for models 2 and 3. In these specifications no change and one instantaneous structural change are

[3] Enders and Granger (1998) proposed unit root tests for two regime TAR model. They named the linear trend as the linear attractor. Hence, following their suggestion we called this nonlinear trend as a nonlinear attractor.

limiting cases, whereas this unit root specification which we have given in equations 1, 2 and 3 is more general, since it covers gradual structural changes as well[4].

We establish the hypotheses for unit root testing based on equations 1, 2 and 3 as follows:

$H_0 : Unit \quad Root$, (Linear Nonstationary)

$H_a : Nonlinear \quad Stationary$ (Nonlinear and Stationary around smoothly changing trend and intercept) (5)

Following Leybourne *et al.* (1996) the test statistics proposed here are calculated with a two-step procedure:

Step 1. Using a nonlinear least squares (NLS) algorithm, estimate only deterministic component of the preferred model and compute the NLS residuals

Model 1 $\hat{\varepsilon}_t = y_t - \hat{\alpha}_1 - \hat{\alpha}_2 S_t(\gamma, \tau)$

Model 2 $\hat{\varepsilon}_t = y_t - \hat{\alpha}_1 - \hat{\beta}_1 t - \hat{\alpha}_2 S_t(\gamma, \tau)$

Model 3 $\hat{\varepsilon}_t = y_t - \hat{\alpha}_1 + \hat{\beta}_1 t + \hat{\alpha}_2 S_t(\gamma, \tau) + \hat{\beta}_2 t S_t(\gamma, \tau)$

Step 2. Compute the KSS statistic, the t ratio associated with $\hat{\rho}_i$ in the ordinary least squares (OLS) regression

$$\Delta\hat{\varepsilon}_t = \hat{\rho}\hat{\varepsilon}_t^3 + \sum_{j=1}^{k} \hat{\delta}_j \Delta\hat{\varepsilon}_{t-j} + \hat{\eta}_t \qquad (6)$$

For model 1, 2 and 3 we denote the t statistics for $\hat{\rho}_i$ as $\overline{t}_{br1}$, $\overline{t}_{br2}$, and $\overline{t}_{br3}$, respectively.

$H_0 : \rho = 0$, for all i, (Linear Nonstationary)

$H_0 : \rho < 0$, for some i, (Nonlinear and Stationary around nonlinear trend and intercept)

Table 1. Critical Values

	Model 1			**Model 2**			**Model 3**		
	%10	%5	%1	%10	%5	%1	%10	%5	%1
25	-3.691	-4.133	-5.056	-4.296	-4.728	-5.543	-4.609	-5.048	-5.873
50	-3.521	-3.870	-4.571	-3.963	-4.327	-5.106	-4.214	-4.593	-5.380
100	-3.509	-3.821	-4.443	-3.889	-4.202	-4.777	-4.090	-4.411	-5.041
200	-3.496	-3.810	-4.424	-3.885	-4.189	-4.771	-4.062	-4.382	-4.980
500	-3.489	-3.801	-4.412	-3.879	-4.180	-4.757	-4.053	-4.370	-4.969

[4] For further discussion and the possible extensions see Leybourne *et al.* (1996).

2.1 Finite sample performance

We have investigated the empirical power of our newly proposed test by using the following data generating process where the process is a stationary nonlinear adjustment around a smooth transition from one constant value to another. Thus, the following ST-ESTAR(1) was employed as a DGP:

$$y_t = \alpha_1 + \alpha_2 S_t(\gamma,\tau) + \varepsilon_t \ , \quad \mu_0 = 0$$

$$\Delta\varepsilon_t = \alpha + \gamma\varepsilon_{t-1}\left(1 - \exp\left[-\theta\varepsilon_{t-1}^2\right]\right) + \eta_t \ , \qquad \eta_t \sim NID(0,1)$$

where S_t is defined as before, and all combinations of the following parameter values were used; two extreme values for the gamma parameter $\gamma = -0.1, -1.0$, two extreme values for the transition speed parameter $\theta = 0.01, 1.0$ and small and large values for the structural break parameter $\alpha_2 = 2.0, 10.0$. The results from these power experiments for a sample size of $T = 100$ are given in Table 2.1.1.

Table2.1.1 The power comparison of alternative tests

α_2	λ	c	θ	γ	$S_{\alpha,NL}$	s_α	t_{s_α}	F_α	t_{NL}	$T_{\max_t}$	Φ_t	τ_μ
2.0	0.5	0.2	0.01	-0.1	0.062	0.046	0.046	0.034	0.062	0.064	0.048	0.044
2.0	0.5	0.2	1.0	-0.1	0.086	0.076	0.070	0.068	0.190	0.144	0.162	0.096
2.0	0.5	0.2	0.01	-1.0	0.164	0.130	0.148	0.116	0.234	0.176	0.208	0.188
2.0	0.5	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.0	0.5	0.5	0.01	-0.1	0.070	0.064	0.064	0.052	0.086	0.080	0.074	0.040
2.0	0.5	0.5	1.0	-0.1	0.100	0.092	0.084	0.086	0.148	0.110	0.134	0.100
2.0	0.5	0.5	0.01	-1.0	0.152	0.132	0.110	0.108	0.208	0.152	0.180	0.152
2.0	0.5	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.0	5.0	0.2	0.01	-0.1	0.050	0.038	0.042	0.026	0.078	0.068	0.068	0.040
2.0	5.0	0.2	1.0	-0.1	0.116	0.110	0.092	0.098	0.206	0.142	0.174	0.100
2.0	5.0	0.2	0.01	-1.0	0.160	0.130	0.126	0.124	0.204	0.174	0.190	0.142
2.0	5.0	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.0	5.0	0.5	0.01	-0.1	0.052	0.042	0.040	0.032	0.092	0.060	0.074	0.060
2.0	5.0	0.5	1.0	-0.1	0.096	0.118	0.088	0.098	0.200	0.120	0.164	0.082
2.0	5.0	0.5	0.01	-1.0	0.190	0.162	0.130	0.150	0.236	0.164	0.212	0.174
2.0	5.0	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10.0	0.5	0.2	0.01	-0.1	0.026	0.012	0.016	0.008	0.006	0.020	0.002	0.018
10.0	0.5	0.2	1.0	-0.1	0.076	0.068	0.052	0.054	0.006	0.020	0.000	0.022
10.0	0.5	0.2	0.01	-1.0	0.120	0.080	0.060	0.074	0.016	0.026	0.012	0.024
10.0	0.5	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	0.008	0.038	0.002	0.231
10.0	0.5	0.5	0.01	-0.1	0.042	0.042	0.040	0.034	0.014	0.010	0.004	0.014
10.0	0.5	0.5	1.0	-0.1	0.114	0.088	0.068	0.072	0.014	0.012	0.004	0.012
10.0	0.5	0.5	0.01	-1.0	0.242	0.161	0.191	0.111	0.030	0.020	0.010	0.030
10.0	0.5	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	0.072	0.058	0.042	0.318
10.0	5.0	0.2	0.01	-0.1	0.114	0.028	0.034	0.016	0.010	0.032	0.002	0.066
10.0	5.0	0.2	1.0	-0.1	0.144	0.074	0.068	0.068	0.004	0.028	0.004	0.076
10.0	5.0	0.2	0.01	-1.0	0.188	0.070	0.086	0.064	0.004	0.024	0.000	0.064
10.0	5.0	0.2	1.0	-1.0	1.000	1.000	1.000	1.000	0.004	0.036	0.002	0.434
10.0	5.0	0.5	0.01	-0.1	0.160	0.060	0.096	0.062	0.026	0.032	0.018	0.064
10.0	5.0	0.5	1.0	-0.1	0.274	0.148	0.180	0.144	0.016	0.024	0.008	0.052

10.0	5.0	0.5	0.01	-1.0	0.398	0.190	0.198	0.182	0.020	0.032	0.014	0.068
10.0	5.0	0.5	1.0	-1.0	1.000	1.000	1.000	1.000	0.058	0.070	0.038	0.282

Note 1: Leybourne *et al.* (1996) stated that Model 1's natural competitor is the ADF test that involves both an intercept and trend term. Therefore, we compare the Model 1 proposed in this paper with the KSS, ADF and EG tests.

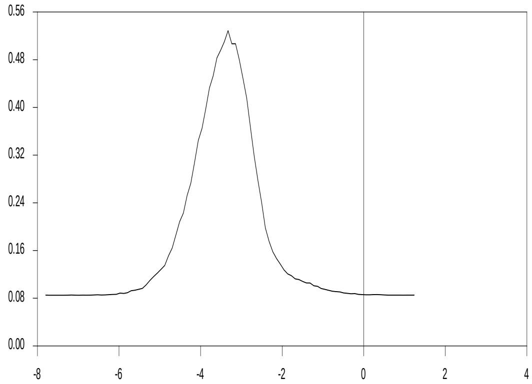
Note 2 : $S_{\alpha,NL}$, s_{α} , $t_{s_{\alpha}}$ and F_{α} denote the proposed test, LNV, Solis max-t and F tests, respectively. t_{NL} , $T_{\max_t}$, Φ_t and τ_{μ} denote the KSS, EG max-t, EG F and DF tests, respectively. The second group of tests does not cover the structural break in their testing procedure.

For a small break ($\alpha_2 = 2.0$), the power of the KSS test exceeds that of the newly proposed test. However, as expected, in the large break case **(**$\alpha_2 = 10.0$) the newly proposed test over performs all of the tests in all parameter regions.

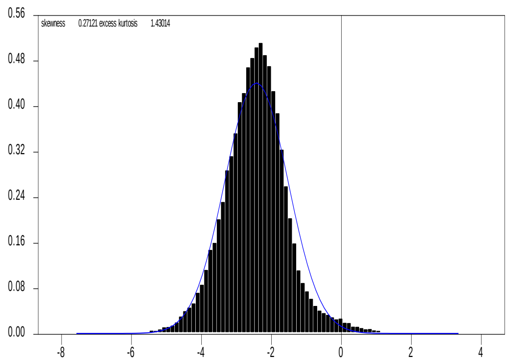
2.2 The Asymptotic Properties of the Proposed Test

Leybourne et al. (1996) state that as NLLS estimation of the parameters gamma (γ) and tou (τ) does not admit closed form solutions, it would be extremely difficult to subsequently establish any analytical relationship between the residual terms that are obtained from the STR estimation of the deterministic component and the dependent variable. Therefore, this makes the determination of the null asymptotic distribution of the test statistics by analytical means more or less intractable. Moreover, in our testing procedure we are introducing another form of nonlinearity around the deterministic component which makes it harder to obtain the asymptotic distribution. Thus, we use the simulation methodology to examine the nonlinear asymptotic relationship. In order to see the difference between normal distribution and the distribution that we obtained from our test statistics, we have obtained density function of the test statistics. The below figures are the simulation results:

The simulated density of test statistics for T= 50



The comparison with normal distribution



Observations	50000
Sample Mean	-2.415
Variance	0.822
Standard Error	0.906
Skewness	0.271 (0.000)
Kurtosis (excess)	1.430 (0.000)
Jarque-Bera	4874.005 (0.000)

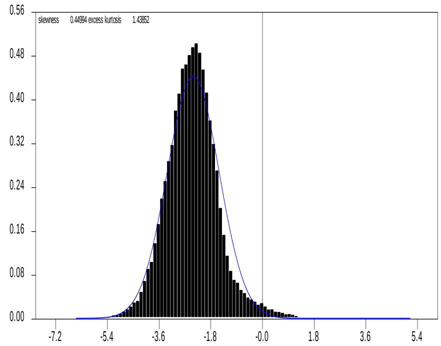
*The solid line is normal distribution and the shaded area is simulated density function of the proposed test

* Number of Simulation 50000

* Time series dimension T = 50

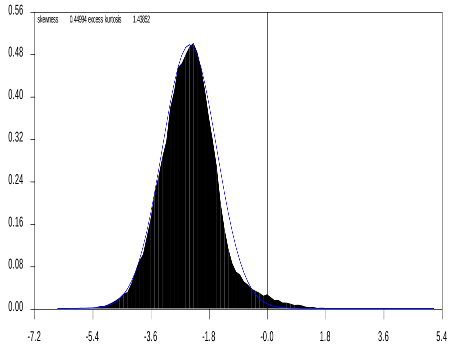
Figure 1. The simulated density of test statistics and comparison with normal distribution for T: 50

The simulated density of test statistics



Sample Mean	-2.400
Variance	0.815
Standard Error	0.903
Skewness	0.449 (0.000)
Kurtosis (excess)	1.438 (0.000)
Jarque-Bera	5998.221 (0.000)

The comparison with normal distribution



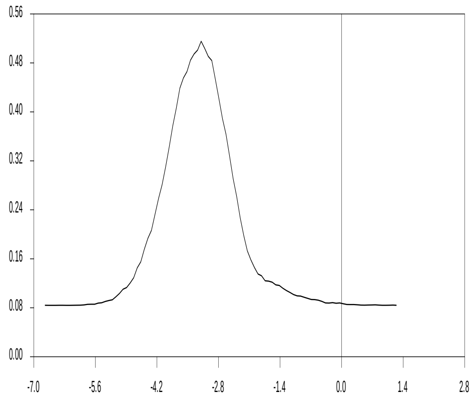
*The solid line is normal distribution and the shaded area is simulated density function of the proposed test

* Number of Simulation 50000

* Time series dimension T = 100

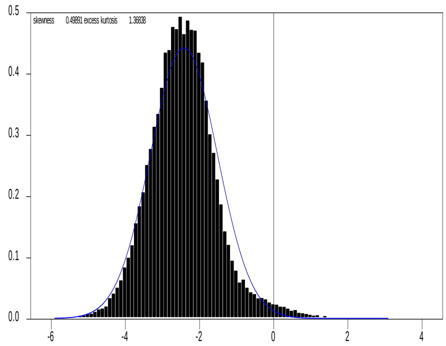
Figure 2. The simulated density of test statistics and comparison with normal distribution for T: 100

The simulated density of test statistics for



Sample Mean	-2.413
Variance	0.821
Standard Error	0.906
Skewness	0.506 (0.000)
Kurtosis (excess)	1.408 (0.000)
Jarque-Bera	6271.965 (0.000)

The comparison with normal distribution



*The solid line is normal distribution and the shaded area is simulated density function of the proposed test

* Number of Simulation 50000

* Time series dimension T = 500

Figure 3. The simulated density of test statistics and comparison with normal distribution for T: 500

3. Empirical Example

In this section we empirically apply all the unit root tests utilized in the power analysis to examine the validity of the purchasing power parity (PPP) hypothesis for Argentina over the period 2003:6-2011:10. Monthly data on the bilateral exchange rate of the national currency against the U.S. dollar and on consumer price indices (CPI) was taken from International Monetary Fund's *International Financial Statistics* (IFS) database. The base year for the CPI is 1997. All variables were put into natural logarithms before the analysis.

Table 11. The PPP hypothesis under alternative unit root tests.

	$S_{\alpha,NL}$	s_{α}	F_{α}	t_{NL}	Φ_t	τ_{μ}
Argentina	-7.315	-2.156	2.325	-2.398	1.813	-1.770

Note: LNV %10 %5 and %1 significance level -3.909, -4.232,-4.882, Sollis %10 %5 and %1 significance level 7.844, 9.191,12.244. KKS %10 %5 and %1 significance level -2.66, -2.93,-3.48. EG %10 %5 and %1 significance level 3.79, 4.64 ,6.57. ADF 10 %5 and %1 significance level -2.58, -2.89, -3.51

The results of the ADF, PP, KSS, EG and Sollis unit root tests recommend that the null hypothesis of a unit root is rejected at the conventional significance levels. These results contradict the PPP hypothesis. On the other hand, our newly proposed test that allows for nonlinear adjustment towards LNV type trend function rejects the null hypothesis of a unit root at 1% significance level, which provides an evidence for the PPP hypothesis. This finding recommends that a model that allows for gradual structural breaks and nonlinear adjustment might be more suitable for the Argentinian RER series.

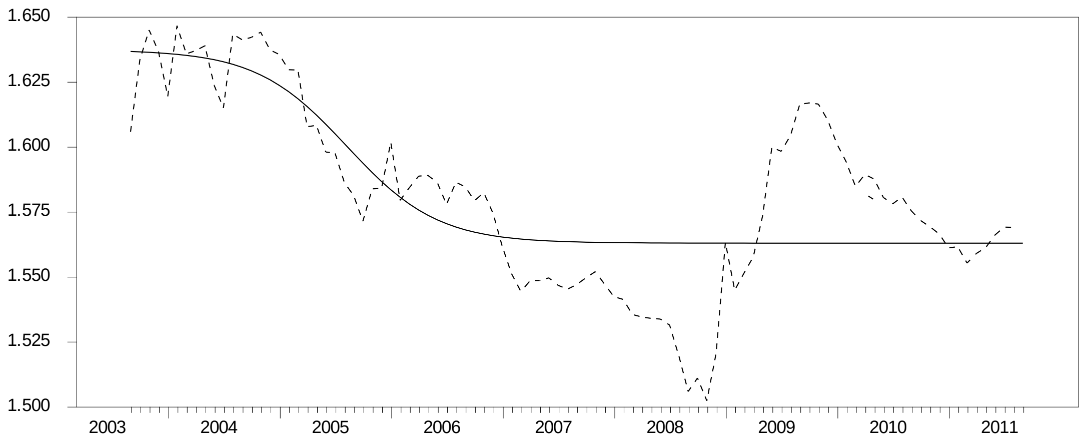


Figure 1. Estimated STR type trend functions (Model A) for Argentina

4. Conclusion

In this study, we have proposed a nonlinear unit root which also considers structural break. By using this newly proposed test we show the validity of the PPP hypothesis for the Argentinian real exchange rate series.

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